

NUMERICAL IMPLEMENTATION OF ICE RHEOLOGY FOR EUROPA'S SHELL. A. C. Barr, R. T. Pappalardo, *Laboratory for Atmospheric and Space Physics, Campus Box 392, University of Colorado, Boulder, CO 80309, (amy.barr@colorado.edu).*

We present a discussion of approximations to the temperature dependent part of the rheology of ice. We have constructed deformation maps using the superplastic rheology of *Goldsby & Kohlstedt, 2001*, and find that the rheologies that control convective flow in the Europa's are likely grain boundary sliding ($Q^*=49$ kJ/mol, $n=1.8$, $p=1.4$) and basal slip ($Q^*=60$ kJ/mol, $n=2.4$) for a range of grain sizes $0.1 \text{ mm} < d < 1 \text{ cm}$. We compare the relative merits of two different approximations to the temperature dependence of viscosity and argue that for temperature ranges appropriate to Europa, implementing the non-Newtonian, lab-derived flow law directly is required to accurately judge the onset of convection in the ice shell and temperature gradient in the near-surface ice.

Deformation Maps: Deformation maps for ice I were constructed using the rheology of *Goldsby & Kohlstedt, 2001*, which expresses a composite flow law for ice I as the sum of four individual flow laws with different dependence upon temperature, strain-rate and grain size. The deformation map shows the locus of points in $\sigma - T$ space where the strain rate from each pair of flow laws are equal. The flow law that contributes the largest strain rate is judged to be dominant in that region. Maps for $d = 0.1 \text{ mm}$ and $d = 1 \text{ cm}$, which bracket common estimates of the grain size within Europa's ice shell [*McKinnon, 1999*], are shown in Figure 1 a,b. For the low ($\sim 0.01 \text{ MPa}$) convective stresses in an ice shell 10's of km thick, the controlling rheology of the ice shell is grain boundary sliding, but basal slip can become important if the grain size is small.

Implementing Temperature Dependence: There is a common method of approximating temperature-dependent rheology used by the Earth mantle convection community [e.g. *Solomatov, 1995*] and applied to icy satellite convection [*McKinnon, 1999; Nimmo & Manga, 2002*] which approximates the lab-derived flow law ($\eta \sim \exp(Q^*/RT)$) as $\eta \sim \exp(-\gamma T)$ where γ is the Frank-Kamenetskii (FK) parameter:

$$\gamma = \left. \frac{\partial(\ln \eta)}{\partial T} \right|_{T_i} \quad (1)$$

Here T_i is the interior temperature of the ice shell, which is not known *a priori*, but is typically close to the melting point (T_m) so it is assumed that $\gamma = Q^*/nRT_m^2$. The viscosity contrast across the convecting layer $\Delta\eta = \exp(E\Delta T)$. Use of this approximation results in lower surface viscosities than predicted by the lab-derived flow law. This does not affect the outcome of stagnant-lid (large $\Delta\eta$) convection simulations, provided the resulting $\Delta\eta$ is high enough that the surface ice remains immobile ($\Delta\eta > 10^4$). For grain boundary sliding within Europa's ice shell, $\gamma\Delta T \sim 7.75$ and $\Delta\eta = 5 \times 10^3$. This relatively low $\Delta\eta$ results in a sluggish lid convection, where the top-most layer of cold ice has a low enough viscosity to participate in convective flow by being dragged along the surface of the convecting region.

An alternative way to approximate the temperature depen-

dence of ice viscosity to use a best-fit, temperature-linearized flow law of form [*Reese, et al., 1999*]:

$$\eta(T) = b \exp(-\bar{E}T) \quad (2)$$

where the parameters \bar{E} and b are determined from a least-squares fit to the lab-derived flow law. For grain boundary sliding in the European temperature range $T_s=100 \text{ K}$ to $T_m=260 \text{ K}$, the best-fit line underestimates the lab-derived flow law by a factor of $\sim 10 - 100$ at the base of the ice shell. This does not change the physics of the stagnant lid convection within the ice shell, which is most sensitive to the viscosity at the base of the rheological lithosphere, not at the base of the ice shell [*Reese, et al., 1999*]. It does have a large effect the critical thickness at which the ice shell begins to convect. For example, an artificial underestimate of viscosity at T_m by a factor of ~ 100 results in an underestimate of critical shell thickness for the onset of convection of order $(100)^{1/3} \sim 5$.

A plot comparing these approximations to $\eta(T)$ and the lab-derived flow law is shown in Figure 1c.

Comparison Between Rheological Approximations and Lab-Derived Law: We performed three numerical simulations using the Citcom finite element software [*Moresi and Gurnis, 1996; Zhong, et al., 1998; Zhong, et al., 2000*] in order to compare the accuracy of these two approximations. All three simulations had a Rayleigh number of 2×10^6 and constant temperature boundary conditions with $T_s=100 \text{ K}$ and $T_m=260 \text{ K}$, appropriate for Europa's ice shell. For grain boundary sliding with a grain size of 1 mm and strain rate of 10^{-10} s^{-1} , this corresponds to an ice shell thickness of $\sim 20 \text{ km}$. Since the temperature dependence of ice rheology is of key interest in this test, we did not implement the strain-rate dependence of viscosity.

Results from implementing the lab-derived flow law directly are shown in Figure 1d. An effective $\gamma\Delta T$ for this rheology was calculated using the T_i value obtained in this simulation, $T_i = 245 \text{ K}$, which implies $\gamma\Delta T = 8.68$.

Results from implementing a temperature-linearized simulation with $\gamma\Delta T = 8.68$ are shown in Figure 1e. The viscosity contrast across the convecting layer implied by this rheology $\Delta\eta \sim 6 \times 10^3$, low enough to permit sluggish lid convection. As a result, the heat flow through the ice shell (Nu) in this simulation was 50% higher than the value obtained implementing the lab-derived flow law. The thickness of the stagnant lid δ_o , which scales roughly as $1/Nu$, is underestimated by 40% compared to the lab-derived law.

Results from implementing the best-fit linearized flow law are shown in Figure 1f. The viscosity contrast across the ice shell implied by the best-fit line is $\Delta\eta \sim 10^8$, predicting stagnant lid behavior. The heat flow through the ice shell agrees with the lab-law at the 10% level. Estimates of the interior temperature (T_i) are correct to within $>5\%$ in either approximation.

Discussion: If a temperature-linearized rheology must be

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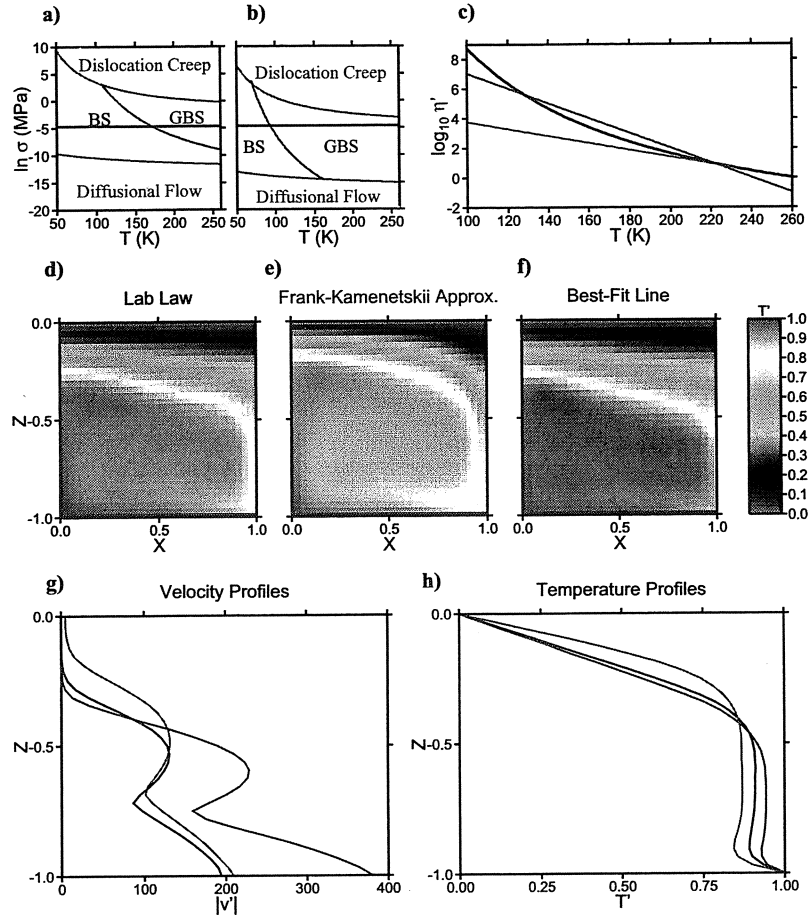


Figure 1: a) Deformation map for Ice I with grain size $d = 0.1$ mm. b) Deformation map for Ice I with grain size $d = 1$ cm. Based on the rheology of Goldsby & Kohlstedt, 2001. BS=basal slip, GBS=grain boundary sliding. Purple horizontal line indicates approximate level of convective stress in Europa's ice shell, ~ 0.01 MPa. c) Approximations to the temperature dependent portion of the GBS rheology for $d = 1$ mm within Europa's ice shell. Black: lab-derived flow law; Red: Frank-Kamenetskii approximation (FK) (eq. 1); Blue: best-fit, temperature-linearized law (eq. 2). Relative viscosity values (η') are scaled to the viscosity at the melting point of ice at $T = 260$ K predicted by the GBS flow law. d) Isotherms calculated using the lab-derived GBS rheology with $Ra = 2 \times 10^6$. e) Isotherms calculated using an effective FK parameter (eq. 1) to approximate the GBS rheology. f) Isotherms calculated using a best-fit, temperature-linearized approximation of GBS rheology. g) Velocity profiles from the above simulations. The simulation employing an effective FK parameter has a non-zero surface velocity and is therefore in the sluggish lid regime of behavior due to the low $\Delta\eta$ predicted by this approximation. h) Temperature profiles from the above simulations. Both approximations to the lab-law predict the value of T_i to within a few percent.

used for numerical reasons, or to calculate bulk parameters of the ice shell using pre-existing scaling laws, using a best-fit linear flow law rather than the standard Frank-Kamenetskii approximation can provide more accurate values for the heat flux (Nu) and the stagnant lid thickness (δ_o). In order to accurately capture the overall behavior of the ice shell, the appropriate creep regime of the lab-derived flow law should be implemented directly in numerical simulations. We are currently in the process of implementing this form of temperature dependence in conjunction with a fully strain-rate dependent ice rheology to understand the geological and astrobiological

consequences of convection within Europa's ice shell.

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